AP Calculus BC

Infinite Series

The Integral and p-test

Name _____

Use the Integral Test to determine the convergence or divergence of the series.

1. $\sum_{n=1}^{\infty} \frac{1}{n+1}$

Diverges

2. $\sum_{n=1}^{\infty} \frac{1}{3n+5}$

Diverges

3. $\sum_{n=1}^{\infty} e^{-n}$

Converges

4. $\sum_{n=1}^{\infty} ne^{-n/2}$

Converges

5. $\frac{1}{2}+\frac{1}{5}+\frac{1}{10}+\frac{1}{17}+\frac{1}{26}+\ldots$

Converges

6. $\frac{1}{3}+\frac{1}{5}+\frac{1}{7}+\frac{1}{9}+\frac{1}{11}+\ldots$

Diverges

7. $\frac{\ln 2}{2}+\frac{\ln 3}{3}+\frac{\ln 4}{4}+\frac{\ln 5}{5}+\frac{\ln 6}{6}+\ldots$

Diverges

8. $\frac{\ln 2}{\sqrt{2}}+\frac{\ln 3}{\sqrt{3}}+\frac{\ln 4}{\sqrt{4}}+\frac{\ln 5}{\sqrt{5}}+\frac{\ln 6}{\sqrt{6}}+\ldots$

Diverges

9. $\frac{1}{\sqrt{1}\left(\sqrt{1}+1\right)}+\frac{1}{\sqrt{2}\left(\sqrt{2}+1\right)}+\frac{1}{\sqrt{3}\left(\sqrt{3}+1\right)}+\ldots\frac{1}{\sqrt{n}\left(\sqrt{n}+1\right)}+\ldots$

Diverges

10. $\frac{1}{4}+\frac{2}{7}+\frac{3}{12}+\ldots+\frac{n}{n^2+3}+\ldots$

Diverges

11. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}}$

Diverges

12. $\sum_{n=2}^{\infty} \frac{\ln n}{n^3}$

Converges

13. $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$

Diverges

14. $\sum_{n=1}^{\infty} \frac{\arctan n}{n^2+1}$

Converges

15. $\sum_{n=1}^{\infty} \frac{n}{n^4+1}$

Converges

Explain why the Integral Test does not apply to the series.

16. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

a_n is not positive for all n

17. $\sum_{n=1}^{\infty} e^{-n} \cos n$

a_n is not positive for all n

18. $\sum_{n=1}^{\infty} \frac{2+\sin n}{n}$

a_n is not decreasing for all n

19. $\sum_{n=1}^{\infty} \left(\frac{\sin n}{n}\right)^2$

a_n is not decreasing for all n

Use the p-test to determine the convergence or divergence of the series.

20. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}}$

Diverges

21. $\sum_{n=1}^{\infty} \frac{1}{n^2}$

Converges

22. $\sum_{n=1}^{\infty} \frac{1}{n^{5/3}}$

Converges

23. $\sum_{n=1}^{\infty} \frac{1}{n^{\pi}}$

Converges

24. $1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{4}}+\ldots$

Diverges

25. $1+\frac{1}{4}+\frac{1}{9}+\frac{1}{25}+\ldots$

Converges

26. $1+\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}+\frac{1}{4\sqrt{4}}+\frac{1}{5\sqrt{5}}+\ldots$

Converges

Find a positive value of P for which the series converges.

27. $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$

p > 1

28. $\sum_{n=2}^{\infty} \frac{\ln n}{n^p}$

p > 1

29. $\sum_{n=1}^{\infty} \frac{n}{(1+n^2)^p}$

p > 1

30. $\sum_{n=1}^{\infty} n\left(1+n^2\right)^p$

Diverges for all p